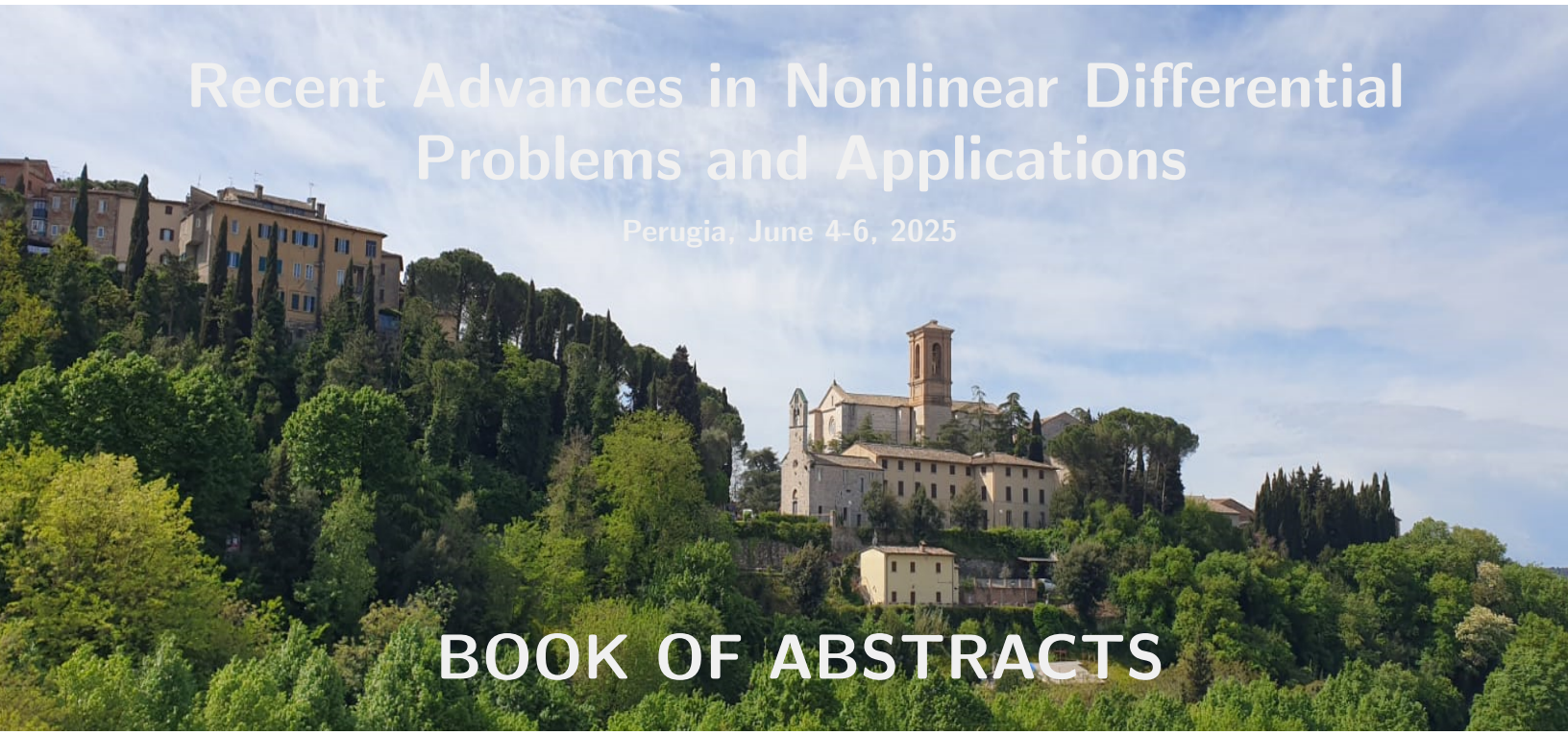
Finanziato dall'Unione europea NextGenerationEU
Ministero dell'Università e della Ricerca
Italiadomani PIANO NAZIONALE DI RIPRESA E RESILIENZA
A.D. 1308 unipg UNIVERSITÀ DEGLI STUDI DI PERUGIA

PRIN 2022 - codice progetto: 2022ZXZTN2 “Nonlinear differential problems with applications to real phenomena”



Recent Advances in Nonlinear Differential Problems and Applications

Perugia, June 4-6, 2025

BOOK OF ABSTRACTS

A.D. 1308 unipg
DIPARTIMENTO DI MATEMATICA E INFORMATICA

INdAM GNAMPA

Lamberto Cesari Centro Studi

Città di PERUGIA
con il patrocinio del Comune di Perugia

Table of Contents

Program

June 04, 2025

Morning				
11:00	12:30	***Registration***		
Afternoon				
14:00	14:30	***Opening Ceremony***		
14:30	15:15	Gabriele Bonanno	Università di Messina	Some remarks on variational methods ☞ Abstract
15:15	16:00	Helene Frankowska	Sorbonne Université, Paris 6	Differential Inclusions under State Constraints in Wasserstein Spaces ☞ Abstract
16:00	16:30	***Coffee Break***		
16:30	17:00	Salvatore A. Marano	Università di Catania	Some recent results on singular Dirichlet problems ☞ Abstract
17:00	17:30	Alessandro Fonda	Università di Trieste	TBA ☞ Abstract
17:30	18:00	Pasquale Candito	Università di Palermo	Existence and multiplicity results for a class of parametric quasilinear elliptic problems ☞ Abstract

June 05, 2025

Morning				
09:00	09:45	Wojciech Kryszewski	Politechnika Łódzka	A generalized infinite-dimensional Hopf-Poincaré formula ☞ Abstract
09:45	10:30	Radu Precup	Babeş-Bolyai University and Tiberiu Popoviciu Institute of Romanian Academy	Kirchhoff-type parabolic equations with nonlocal in space and time diffusion coefficients ☞ Abstract
10:30	11:00	***Coffee Break***		
11:00	11:30	Gennaro Infante	Università della Calabria	Birkhoff-Kellogg type results with applications ☞ Abstract
11:30	12:00	Roberto Livrea	Università di Palermo	Positive solutions for quasilinear nonlocal Dirichlet problems ☞ Abstract
12:00	12:30	Valeria Marraffa	Università di Palermo	Relaxation result for differential inclusions with Stieltjes derivative ☞ Abstract
12:00	12:30	Giuseppe Viglialoro	Università di Cagliari	Considerations on Keller–Segel models with positive total flux ☞ Abstract
Lunch				
Afternoon				
14:30	15:15	Zuzana Dosla	Masarykova Univerzita	Radial solutions to equation with regularly varying operator ☞ Abstract
15:15	15:45	Luisa Malaguti	Università di Modena – Reggio Emilia	Wavefronts for a degenerate reaction-diffusion system with application to bacterial growth models ☞ Abstract
15:45	16:15	***Coffee Break***		
16:15	16:35	Eleonora Amoroso	Università di Messina	Multiple solutions for nonlinear p-Laplacian problems in the whole space ☞ Abstract
16:35	16:55	Andrei Stan	Babeş-Bolyai University and Tiberiu Popoviciu Institute of Romanian Academy	Localization of critical points in conical sets via the method of Nehari manifolds ☞ Abstract
16:55	17:15	Umberto Guarnotta	Università Politecnica delle Marche	A p-Laplacian problem in $\mathbb{R}^N$ with singular, convective, critical reaction ☞ Abstract
Guided Tour of the City Center				
Social Dinner				

June 06, 2025

Morning				
09:00	09:45	Adrian Constantin	Universität Wien	Nonlinear atmospheric flow patterns confined to zonal cloud bands ☞ Abstract
09:45	10:30	Fabio Zanolin	Università di Udine	Non-negative solutions of a Dirichlet problem with a non-Lipschitzian term ☞ Abstract
10:30	11:00	Maria Patrizia Pera	Università di Firenze	Global Bifurcation of Forced Oscillations of ODE's involving the Φ-Laplacian ☞ Abstract
11:00	11:30	***Coffee Break***		
11:30	12:00	Cristina Marcelli	Università Politecnica delle Marche	Front propagation in reaction-diffusion-convection equations with discontinuous coefficients ☞ Abstract
12:30	13:00	Serena Matucci	Università di Firenze	A unifying approach to the uniform asymptotic stability and exponential stability of some applied sciences models ☞ Abstract
12:30	13:00	Annamaria Barbagallo	Università di Napoli "Federico II"	Levitin-Polyak well-posedness of stochastic variational inequalities and applications ☞ Abstract
13:00	13:20	***Closing***		

Book of Abstracts

Multiple solutions for nonlinear p-Laplacian problems in the whole space

Eleonora Amoroso

University of Messina, Italy
eleonora.amoroso@unime.it

In this talk, we discuss some existence and multiplicity results for nonlinear p-Laplacian equations defined in the whole space $\mathbb{R}^N$. In particular, we deal with the following parametric problem

$$-\Delta_p u + a(x)|u|^{p-2}u = \lambda f(x,u) \quad \text{in} \quad \mathbb{R}^N, \qquad u \in W^{1,p}(\mathbb{R}^N), \tag{P_λ}$$

and we provide sufficient conditions for the existence of multiple solutions. In particular, two non-zero solutions with opposite energy sign for equations having combined effects of concave and convex nonlinearities are obtained. The approach is based on variational methods and critical points theory.

back to Program

Levitin-Polyak well-posedness of stochastic variational inequalities and applications

Annamaria Barbagallo

Department of Mathematics and Applications "R. Caccioppoli", University of Naples Federico II, Naples, Italy
annamaria.barbagallo@unina.it

The talk aims to study the Levitin-Polyak well-posedness (shortly, LP well-posedness) of stochastic variational inequalities. A characterization of the LP well-posedness is obtained considering the size of LP approximating solution sets. The equivalence between the LP well-posedness of a stochastic variational inequality with the existence and uniqueness of the solution is proved. In addition, the LP well-posedness in the generalized sense is characterized. Finally, the theoretical results are applied to the spatial price equilibrium problem in which the supply and demand functions are available and are functions of the supply and demand prices, respectively, and, in addition, the data are subject to random fluctuations.

References

[1] Barbagallo, A.; *Levitin-Polyak well-posedness of stochastic variational inequalities and applications to a random spatial price equilibrium problem*, submitted.

☜ back to Program

Some remarks on variational methods

Gabriele Bonanno

University of Messina, Italy
gabriele.bonanno@unime.it

☜ back to Program

Existence and multiplicity results for a class of parametric quasilinear elliptic problems

Pasquale Candito

Department of Mathematics and Computer Science, University of Palermo, Italy
pasquale.candito@unipa.it

The purpose of this talk is to present some recent existence and multiplicity results for a class of parametric quasilinear elliptic problems, with particular attention to those contained in the papers mentioned below. Roughly speaking, our approach is based on the synergy between a local minimum theorem and the mountain pass lemma.

References

[1] Candito, P., Guarnotta U., Perera, K.; *Two solutions for a parametric singular $p-$Laplacian problem*, J. Nonlinear Var. Anal. 4 (2020), No. 3, pp. 455-468.

[2] Candito, P., Guarnotta, U., Livrea, R.; *Existence of two solutions for singular Φ-Laplacian problems*, Adv. Nonlinear Stud. 22 (2022), no. 1, 659–683.

[3] Candito, P., Failla, G., Livrea, R.; *Positive solutions for a p-Laplacian equation with sub-critical singular parametric reaction term*, Z. Anal. Anwend., to appear.

[4] Candito, P., Carl, S., Livrea, R.; *Critical points in open sublevels and multiple solutions for parameter-depending quasilinear elliptic equations*, Adv. Differential Equations, 19 (2014), 1021–1042.

☜ Back to Program

Nonlinear atmospheric flow patterns confined to zonal cloud bands

Adrian Constantin

University of Vienna
adrian.constantin@univie.ac.at

Motivated by the dynamics of Jupiter's Great Red Spot and Saturn's polar hexagon, featuring oscillations superimposed on a mean current, we derive some exact solutions of the nonlinear governing equations for atmospheric flow confined to zonal cloud bands on the gas giants of our solar system. A Hamiltonian approach provides an accurate description of the particle paths, capturing the main observed features of these spectacular atmospheric flow patterns.

☜ back to Program

Radial solutions to equation with regularly varying operator

Zuzana Došlá [1], Mauro Marini [2] and Serena Matucci [3]

[1] Masaryk University, Brno, Czech Republic
dosla@math.muni.cz
[2] University of Florence, 50139 Florence, Italy
mauro.marini@unifi.it
[3] University of Florence, 50139 Florence, Italy
serena.matucci@unifi.it

The existence of radially symmetric solutions outside of a ball for an elliptic differential equation with differential operator in the divergence form with weight leads to the existence of global positive solutions, that is solutions which are positive on the whole half-line $I = [t_0, \infty)$ for the equation

$$(a(t)\Phi(x'))' + b(t)F(x) = 0, \tag{1}$$

where Φ is an increasing odd homeomorphism regularly varying at zero. Many physical problems are modelled by equations involving regularly varying operators, which are nonhomogeneous and/or with domain, or imagine, bounded. We recall, for instance, the mean curvature operators Φ_E, Φ_M in Euclidean and Minkowski spaces, which model fluid mechanics problems and arise in the theory of nonlinear electromagnetism

$$\Phi_E(u) := u/\sqrt{1+u^2}, \quad \Phi_M(u) := u/\sqrt{1-u^2},$$

respectively, their corresponding $p-$generalizations $\Phi_C^{\pm}$ given by

$$\Phi_C^{\pm}(u) := \frac{|u|^{p-2}u}{(1 \pm |u|^p)^{(p-1)/p}}, \quad p > 1,$$

and the operators Φ_{TC} and Φ_{NE}, which arise in torsional creep problems and in nonlinear elasticity phenomena, given by

$$\Phi_{TC}(u) := (|u|^{p-2} + |u|^{n-2})\, u, \ 1 < p < n, \quad \Phi_{NE} := (1+u^2)^p\, u, \ \ p > 1/2,$$

respectively.

The obtained results illustrate how a certain proximity concerning the global positivity and decay of solutions between equation (1) and the half-linear equation

$$(a(t)\Phi_p(x'))' + b(t)\Phi_p(x) = 0,$$

holds, where Φ_p is the p-Laplacian operator $\Phi_p(u) := |u|^{p-2}u, \quad p > 1$.

References

[1] Došlá Z., Marini M., Matucci S., *Zero-convergent solutions for equations with generalized relativistic operator: a fixed point approach*, J. Nonlinear Convex. Anal.

[2] Došlá Z., Marini M., Matucci S., *Global positive solutions for equations with regularly varying operator*, preprint.

back to Program

TBA

Alessandro Fonda

University of Trieste, Italy

☜ Back to Program

Differential Inclusions under State Constraints in Wasserstein Spaces

Hélène Frankowska

CNRS and Sorbonne Université, Paris, France
helene.frankowska@imj-prg.fr

In this talk I will present some viability and invariance theorems in Wasserstein spaces $\mathcal{P}_p(R^d)$ of Borel probability measures with finite $p-$momentum and $p \geq 1$. The differential inclusion is a nonlocal one, described by the transport inclusion

$$\partial_t \mu(t) \in -div\big(F(t, \mu(t))\mu(t)\big) \qquad \mu(0) = \mu_0$$

where $F : R_+ \times \mathcal{P}_p(R^d) \rightsquigarrow Lip(R^d, R^d)$ is a set-valued map whose values are Lipschitz continuous bounded maps from R^d into itself. A subset $Q \subset \mathcal{P}_p(R^d)$ is called viable under F if for every $\tau \geq 0$ and $\mu_\tau \in Q$, there exists a solution $\mu(\cdot)$ of the above differential inclusion such that $\mu(t) \in Q$ for all $t \geq \tau$. Q is called invariant under F if every such solution satisfies $\mu(t) \in Q$ for all times $t \geq \tau$. To characterise these two properties in the spirit of the classical results I will introduce an analogue of the Peano contingent cone to Q.

Two cases will be discussed. When $p > 1$ the obtained results are stronger and proved using using some duality arguments [1]. In contrast for $p = 1$ the Euler approximation scheme is applied [2].

To state the main results I will discuss the extensions to Wasserstein spaces of the Scorza-Dragoni and mean-value theorems to this metric framework.

References

[1] Bonnet B. & Frankowska H.; *Viability and invariance of proper sets for continuity inclusions in Wasserstein spaces*, SIMA, 56 (2024), 2863-2914.

[2] Bonnet B., Corella A. & Frankowska H.; *Viability theorem in 1-Wasserstein space*, to be submitted, 2024.

☜ Back to Program

A p-Laplacian problem in $\mathbb{R}^N$ with singular, convective, critical reaction

Laura Baldelli [1] and Umberto Guarnotta [2]

[1] Departamento de Análisis Matemático, Universidad de Granada, Granada, Spain
labaldelli@ugr.es
[2] Dipartimento di Ingegneria Industriale e Scienze Matematiche, Università Politecnica delle Marche, Ancona, Italy
u.guarnotta@univpm.it

The talk is devoted to the problem

$$\begin{cases} -\Delta_p u = \lambda w(x) f(u, \nabla u) + u^{p^*-1} & \text{in } \mathbb{R}^N, \\ u > 0 & \text{in } \mathbb{R}^N, \\ u(x) \to 0 & \text{as } |x| \to +\infty, \end{cases}$$

where $N \geq 2$, $1 < p < N$, and $\lambda > 0$. The nonlinear term $f : (0, +\infty) \times \mathbb{R}^N \to (0, +\infty)$ is a continuous function which is singular in the first variable and p-sublinear with respect to the second one. The weight $w : \mathbb{R}^N \to (0, +\infty)$ satisfies suitable summability and decay conditions. The problem exhibits several features:

- the perturbation f is singular, i.e., it blows up when the solution vanishes;
- f encompasses also convection terms, that is, depending on the gradient of the solution;
- the 'dominating' reaction term has critical growth;
- the setting is the whole $\mathbb{R}^N$;
- pointwise decay (at infinity) of the solutions is required.

We will present an existence result that combines variational methods, truncation techniques, and concentration compactness arguments, together with set-valued analysis and fixed point theory. In addition, De Giorgi's technique, gradient estimates, and nonlinear regularity theory will be employed to ensure local $C^{1,\alpha}$ regularity of solutions, as well as their pointwise decay at infinity, which is quantitatively estimated via blow-up arguments and a priori estimates.

The result is new even in the non-singular case, also for the Laplacian.

References

[1] Baldelli, L., Guarnotta, U.; *Existence and regularity for a p-Laplacian problem in $\mathbb{R}^N$ with singular, convective, critical reaction*, Adv. Nonlinear Anal., 13 (2024), Paper no. 20240033.

[2] Baldelli, L., Guarnotta, U.; *Decay estimates for solutions to non-autonomous critical p-Laplace problems*, preprint.

☜ back to Program

Birkhoff-Kellogg type results with applications

Gennaro Infante

Università della Calabria, Italy
gennaro.infante@unical.it

We present some classical and recent results of Birkhoff-Kellogg type. We illustrate their applicability in the context of ordinary, functional and partial differential equations subject to local, nonlocal and functional boundary conditions.

Supported by the Research project of MUR - Prin 2022 "Nonlinear differential problems with applications to real phenomena" (Grant Number: 2022ZXZTN2).

References

[1] G. D. Birkhoff and O. D. Kellogg, *Invariant points in function space*, Trans. Amer. Math. Soc., **23** (1922), 96–115.

[2] A. Calamai and G. Infante, *An affine Birkhoff–Kellogg type result in cones with applications to functional differential equations*, Mathematical Methods in the Applied Sciences, **46** (2023), 11897–11905.

[3] A. Calamai and G. Infante, *On fourth order retarded equations with functional boundary conditions: a unified approach*, Discrete Contin. Dyn. Syst. Ser. S, **17** (2024), no. 5-6, 2009–2020.

[4] A. Calamai and G. Infante, *Nontrivial solutions of a parameter-dependent heat flow problem with deviated arguments*, in: Topological Methods for Delay and Ordinary Differential Equations, P. Amster and P. Benevieri (Eds.), Advances in Mechanics and Mathematics, vol. 51, Birkhäuser, Cham, Germany (2024), 141–150.

[5] A. Calamai and G. Infante, *On the solvability of parameter-dependent elliptic functional BVPs on annular-like domains*, Discrete Contin. Dyn. Syst. Ser. B, Early Access (2025), Doi: 10.3934/dcdsb.2024193.

[6] A. Calamai, G. Infante and J. Rodríguez–López, *A Birkhoff–Kellogg type theorem for discontinous operators with applications*, Mediterr. J. Math., **21** (2024) Paper No. 149, 19 pp.

[7] G. Infante, *Eigenvalues of elliptic functional differential systems via a Birkhoff-Kellogg type theorem*, Mathematics, **9** (2021), n. 4.

[8] G. Infante, *On the solvability of a parameter-dependent cantilever-type BVP*, Appl. Math. Lett., **132** (2022), 108090.

[9] G. Infante and P. Lucisano, *Eigenvalues of a third order BVP subject to functional BCs*, Topol. Methods Nonlinear Anal., to appear.

[10] M. A. Krasnosel'skiĭ and L. A. Ladyženskiĭ, *The structure of the spectrum of positive nonhomogeneous operators*, Trudy Moskov. Mat. Obšč, **3** (1954), 321–346.

☜ back to Program

A generalized infinite-dimensional Hopf-Poincaré formula

Wojciech Kryszewski

Politechnika Lódzka

One of the most important results in differential topology (as well as in qualitative algebraic topology and geometry) is the Poincaré-Hopf theorem, which states that if M is a C^1 oriented, closed, m-dimensional manifold (without boundary) and $v : M \to TM$ is a continuous vector field with isolated zeros, then $\sum_{p\in v^{-1}(0)} \mathrm{ind}_p(v) = \chi(M)$, where $\chi(M)$ denotes the Euler characteristic of M, and $\mathrm{ind}_p(v)$ is the (local) index of v at p.

This remarkable result addresses the question: to what extent does the structure of a manifold (for example, its number of holes) constrain the existence of certain functions (such as smooth ones) on it? It has far-reaching consequences – for instance, it immediately implies that every smooth vector field on an even-dimensional sphere must vanish somewhere (the so-called 'Hairy Ball theorem'). On the other hand, it is easy to see that smooth, non-vanishing vector fields do exist on the torus. The theorem also provides yet another, but remarkably short, proof of the Fundamental Theorem of Algebra.

If M is a hypersurface of dimension $m = 2k$ in $\mathbb{R}^{m+1}$, the celebrated Gauss-Bonnet formula states that $2\deg(\gamma) = \chi(M) = 2\nu_m^{-1}\int_M K(x)d\mu_M$, where $\gamma : M \to S^m$ is the Gauss map, $K : M \to \mathbb{R}$ is the Gauss-Kronecker curvature of M, and ν_m os the volume of the unit m-dimensional sphere S^m.

Both the Poincaré-Hopf and Gauss-Bonnet theorems establish deep connections between the topology of a surface and more local geometric properties – namely, the singularities of vector fields and their indices, as well as the (global) curvature.

The problem of generalizing the Poincaré-Hopf theorem (and, to a much lesser ex- tent, the Gauss-Bonnet theorem) to infinite-dimensional settings has been studied by several authors, including McCord, Bartsch and Dancer, Cingolani and Degiovanni, Srzednicki, Mc-Cord and others, and (not very recently) Ćwiszewski and myself. I would like to discuss several aspects of these results, culminating in a formula that relates the fixed-point indices of rest points of a completely continuous semiflow – defined on a (not necessarily locally compact) metric space in the interior of an isolating block B – to to the Euler characteristic of the pair (B, B^-), where B^- is the exit set. As a consequence, a generalized Poincaré-Hopf type formula is obtained for differential equations determined by a Lipschitz perturbation of the generator of a compact C^0 semigroup.

☜ back to Program

Differential Inclusions under State Constraints in Wasserstein Spaces

Roberto Livrea

Department of Mathematics and Computer Science, University of Palermo, Italy
roberto.livrea@unipa.it

We present some recent studies on the existence of multiple pairs of positive solution for a Carrier equation of the following type

$$\begin{cases} -a\left(\int_\Omega u^q dx\right)\Delta_p u = f(x,u) & \text{in } \Omega, \\ u > 0 & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega, \end{cases} \tag{2}$$

where $\Omega \subset \mathbb{R}$ is a smooth domain, $q \geqslant 1$, $1 < p < +\infty$, and $a \in C([0,\infty))$ is a changing sign function having a finite number of positive bumps.

As pointed out for example in [4], nonlocal problems having a structure as (2) can be considered for describing biological models of the population diffusion.

Starting from [5, 6], where $p = 2$ and the reaction term does not depend on x, we show different gradual improvements that, in any case, lead to the existence of multiple pairs of positive solutions for problem (2).

First, a suitable monotonicity condition on $f(x,\cdot)$ is required and the goal is achieved by combining variational methods with topological methods, as well as truncation techniques (see [3, 1]). Next, exploiting also set valued analysis, the multiplicity result is still perfectly preserved without any monotonicity assumption, as shown in [2].

References

[1] P. Candito, G. Failla, R. Livrea, *Pairs of Positive Solutions for a Carrier $p(x)$-Laplacian Type Equation*, Mathematics 2024, 12, 2441.

[2] P. Candito, G. Failla, L. Gasínski, R. Livrea, *Multiple positive solutions for quasilinear nonlocal problem via minimal solution and set-valued analysis*, preprint.

[3] P. Candito, L. Gasínski, R. Livrea, J.R. Santos Júnior, *Multiplicity of positive solutions for a degenerate nonlocal problem with p-Laplacian*, Adv. Nonlinear Anal. **11** (2022), 357–368.

[4] M. Chipot, J.F. Rodrigues, *On a class of nonlocal nonlinear elliptic problems*, RAIRO - Modélisation mathématique et analyse numérique **26** (1992), no. 3, 447–467.

[5] L. Gasínski, J.R. Santos Júnior, *Multiplicity of positive solutions for an equation with degenerate nonlocal diffusion*, Comput. Math. Appl. **78** (2019), 136–143.

[6] L. Gasínski, J.R. Santos Júnior, *Nonexistence and multiplicity of positive solutions for an equation with degenerate nonlocal diffusion*, Bull. Lond. Math. Soc. **52** (2020), no. 3, 489–497.

back to Program

Wavefronts for a degenerate reaction-diffusion system with application to bacterial growth models

Luisa Malaguti [1], Elisa Sovrano [2]

[1] Department of Sciences and Methods for Engineering , University of Modena and Reggio Emilia, Reggio Emilia, Italy
luisa.malaguti@unimore.it
[2] Department of Sciences and Methods for Engineering , University of Modena and Reggio Emilia, Reggio Emilia, Italy
elisa.sovrano@unimore.it

We consider the nonlinear system of two coupled reaction-diffusion equations with degenerate diffusivity:

$$\begin{cases} n_t = & n_{xx} - nb, \qquad t \geq 0,\, x \in \mathbb{R} \\ b_t = & [Dnbb_x]_x + nb \end{cases} \tag{3}$$

where D is a positive diffusion coefficient.

This model, introduced by Kawasaki et al. [1], describes the spatial-temporal dynamics of bacterial colonies $b = b(x,t)$ and nutrients $n = n(x,t)$ on agar plates. Kawasaki et al. [1] provided numerical evidence for the existence of wavefront solutions to (3), leaving the analytical confirmation of these solutions an open problem. Satnoianu et al [4] investigated the simplified system where the first equation in (3) is replaced by $n_t = -nb$ which corresponds to assume that the nutrient does not diffuse; through both numerical and analytical approaches, they indicated that wavefronts exist, for their model, and display "sharp" behavior in the bacteria profile at a critical speed. Muñoz-Hernández et al. [3] generalized the model in [4] by assuming $n_t = -f(n,b)$ as the first equation and did a complete discussion about the existence of wavefront solutions to their model; in particular they showed that the profile of such solutions exhibits "sharp" behavior only at the threshold speed.

In [2] we consider the general model (3) and prove the existence of an infinite family of wavefronts parameterized by their wave speed, which varies on a closed positive half-line, thus confirming the conjecture in [1]. We provide an upper bound for the threshold speed and a lower bound for it when D is sufficiently large. The proofs are based on several analytical tools, including the upper and lower solutions approach, the shooting method and the fixed-point theory in Fréchet spaces, to establish existence, and the central manifold theorem to ascertain uniqueness.

References

[1] Kawasaki, K. and Matsushita, M. and Umeda, T. and Shigesada, N.; *Modeling spatio-temporal patterns generated by Bacillus subtilis*, J. Theor. Biol., 188 (1997), 177–185.

[2] Malaguti, L and E. Sovrano; *Wavefronts for a degenerate reaction-diffusion system with application to bacterial growth models*, submitted, (2024).

[3] Muñoz-Hernández, E. and Sovrano, E. and Taddei, V.; *Coupled reaction-diffusion equations with degenerate diffusivity: wavefront analysis*, accepted on Nonlinearity, (2025).

[4] Satnoianu, R. A. and Maini, P. K. and Garduno, F. S. and Armitage, J. P.; *Travelling waves in a nonlinear degenerate diffusion model for bacterial pattern formation*, Discrete Contin. Dyn. Syst. Ser. B, 1 (2001), 339–362

back to Program

Some recent results on singular Dirichlet problems

Salvatore Angelo Marano

Department of Mathematics and Computer Sciences, University of Catania, Italy
marano@dmi.unict.it

Let Ω be a bounded domain in $\mathbb{R}^N$, $N \geq 2$, with a smooth boundary $\partial\Omega$ and let $h : \Omega \times \mathbb{R}^+ \times \mathbb{R}^N \to \mathbb{R}_0^+$ satisfy Carathéodory's conditions. Consider the homogeneous Dirichlet problem

$$\mathcal{L}u = h(x, u, \nabla u) \ \text{ in } \ \Omega, \quad u > 0 \ \text{ in } \ \Omega, \quad u = 0 \ \text{ on } \ \partial\Omega, \tag{4}$$

where $\mathcal{L}$ denotes a quasi-linear elliptic operator in divergence form, patterned after the p-Laplacian or the (p, q)-Laplacian. Roughly speaking, we say that (4) is singular when

$$\lim_{t \to 0^+} h(x, t, \xi) = +\infty.$$

The literature on singular equations is by now very wide. Both monographs and surveys are already available; see, e.g., [2, 4, 5, 6]. In this talk, two possible new research lines, as well as a few related results, will be presented. Precisely,

- singular Dirichlet problems with *highly discontinuous reaction* [3];
- *fractional* equations having *singular and non-locally convective reaction* [1].

References

[1] L. Gambera and S.A. Marano, *Fractional Dirichlet problems with singular and non-locally convective reaction,* arXiv:2411.11026v2 [math.AP]

[2] M. Ghergu and V.D. Radulescu, *Singular elliptic problems: bifurcation and asymptotic analysis*, Oxford Lecture Ser. Math. Appl. **37**, Oxford Univ. Press, Oxford, 2008.

[3] U. Guarnotta and S.A. Marano, *Strong solutions to singular discontinuous p-Laplacian problems,* arXiv:2407.20971v1 [math.AP]

[4] U. Guarnotta, R. Livrea, and S.A. Marano, *Some recent results on singular p-Laplacian equations*, Demonstratio Math. **55** (2022), 415–428.

[5] J. Hernández and F.J. Mancebo, *Singular Elliptic and Parabolic Equations,* in: M. Chipot and P. Quittner (eds), Handbook of Differential Equations, vol. 3, pp. 317–400, Elsevier, Amsterdam, 2006.

[6] F. Oliva and F. Petitta, *Singular elliptic PDEs: an extensive overview,* arXiv:2409.00482v2 [math.AP].

☜ Back to Program

Front propagation in reaction-diffusion-convection equations with discontinuous coefficients

Cristina Marcelli

Università Politecnica delle Marche, Italy
c.marcelli@staff.univpm.it

In this joint paper with Umberto Guarnotta we study the existence and the properties of traveling wave solutions (t.w.s.) for the monostable reaction-diffusion-convection equation

$$f(v)v_x + g(v)v_t = (d(v)v_x)_x + h(v), \quad v(t,x) \in [0,1]$$

where the coefficients d, f, g, h are allowed to have a finite number of jump discontinuities. After giving a suitable definition of t.w.s., we prove the existence of a threshold wave speed c^* such that there exist t.w.s. with speed c if and only if $c \geq c^*$. Moreover, we also provide an estimate for the speed c^* which extend the well down estimate to the present framework with discontinuous coefficients.

back to Program

Relaxation result for differential inclusions with Stieltjes derivative

Valeria Marraffa

Dipartimento di Matematica e Informatica, Università degli Studi, Palermo, Italy
valeria.marraffa@unipa.it

Applying a recent Filippov Lemma for measure differential inclusions, we first present on a compact interval a Filippov-Ważewski-type theorem for the very general setting of differential inclusions involving the Stieltjes derivative with respect to a non-decreasing, left-continuous map g. The solutions $y : [0,1] \to \mathbb{R}^d$ of the relaxed problem

$$y'_g(t) \in \begin{cases} \overline{co}F(t, y(t)), & t \notin D_g, \\ F(t, y(t)), & t \in D_g \end{cases} \qquad y(0) = \xi_0 \tag{5}$$

can be approximated by solutions $z : [0,1] \to \mathbb{R}^d$ of

$$z'_g(t) \in F(t, z(t)), \qquad z(0) = \xi_0, \tag{6}$$

where D_g is the set of discontinuity points of g.
Using the relaxation result on a compact interval we get in the second part a relaxation theorem on an infinite domain. In this case the approximation can be achieved once we allow to the initial value to differ (but remaining close to) from the initial value of the considered solution of the relaxed inclusion. New relaxation results can be deduced for generalized differential problems, for impulsive differential inclusions with multivalued impulsive maps and possibly countable impulsive moments and also for dynamic inclusions on time scales.

References

[1] Marraffa,V. and Satco, B; *Relaxation result for differential inclusions with Stieltjes derivative*, J. Math. Anal. Appl., (2023), 528(2), 127533.

[2] Marraffa,V. and Satco, B; *Relaxation theorem for Stieltjes differential inclusions on infinite intervals*, Mediterr. J. Math. (2023) 20:331.

☜ back to Program

A unifying approach to the uniform asymptotic stability and exponential stability of some applied sciences models

Serena Matucci

University of Florence, Italy
serena.matucci@unifi.it

Many mathematical models in the natural and applied sciences are described by integro-differential equations to take into account memory effects, and can also be potentially subject to impulsive perturbations. Through two examples - one from population dynamics and another from robotics - we show how these models can be studied in a unified framework by reformulating them as abstract Cauchy problems in an appropriate function space.

By applying topological methods from nonlinear analysis and the theory of C_0-semigroups, we establish conditions for the existence, uniqueness, and uniform asymptotic stability of solutions, as well as for the exponential stability of the zero solution, when applicable.

The results presented were obtained in collaboration with Tiziana Cardinali and Paola Rubbioni.

References

[1] Cardinali, T.; Matucci, S.; Rubbioni, P.; Uniform asymptotic stability of a PDE's system arising from a flexible robotics model, *Authorea.* December 05, 2024.

[2] Cardinali, T.; Matucci, S.; Rubbioni, P.; A unifying approach to the uniform asymptotic stability of certain classes of applied sciences models, preprint.

☜ back to Program

Global Bifurcation of Forced Oscillations of ODE's involving the Φ-Laplacian

Alessandro Calamai [1], Maria Patrizia Pera [2] and Marco Spadini [2]

[1] Dipartimento di Ingegneria Civile, Edile e Architettura, Università Politecnica delle Marche
calamai@dipmat.univpm.it
[2] Dipartimento di Matematica e Informatica "Ulisse Dini", Università di Firenze
mpatrizia.pera@unifi.it, marco.spadini@unifi.it

A class of parametric perturbed implicit ordinary differential equations with a generalized Φ-Laplacian type term is considered, allowing the perturbation to be of Carathéodory type regularity. Under suitable assumptions based on Brouwer topological degree, we obtain global bifurcation of nontrivial periodic solutions having the same period as that of the perturbation and emanating from the set of stationary solutions.

References

[1] Calamai, A.; Pera, M.P.; Spadini, M., *Forced oscillations for generalized Φ-Laplacian equations with Carathéodory perturbations*, preprint.

[2] Benevieri, P.; Feltrin, G., *Atypical bifurcation for periodic solutions of Φ-Laplacian systems*, arXiv:2406.00325v1, June 2024.

[3] Furi, M.; Pera M.P., *Carathéodory periodic perturbations of the zero vector field on manifolds*, Topological methods in nonlinear analysis **10** (1997), 79–92.

[4] Calamai, A.; Spadini, M., *Carathéodory periodic perturbations of degenerate systems*, Electronic Journal of Differential Equations, art. no. 39 (2024).

[5] Calamai, A.; Pera, M.P.; Spadini, M., *Branches of forced oscillations for a class of implicit equations involving the Φ-Laplacian* in Topological Methods for Delay and Ordinary Differential Equations – With Applications to Continuum Mechanics, P. Amster and P. Benevieri (Eds.), Birkhäuser, Cham, 151–166, 2024.

☜ back to Program

Kirchhoff-type parabolic equations with nonlocal in space and time diffusion coefficient

Radu Precup

Babeș-Bolyai University and Tiberiu Popoviciu Institute of Romanian Academy, Cluj-Napoca, Romania
r.precup@math.ubbcluj.ro

We investigate the existence and uniqueness of weak solutions for Kirchhoff-type parabolic equations with reaction terms. We consider the Cauchy-Dirichlet problem related to such an equation, in a bounded domain, with the diffusion coefficient depending on the solution itself. We target two types of nonlocal diffusion coefficient: diffusion coefficient which is nonlocal only in space, most often considered in literature, and diffusion coefficient which is nonlocal both in space and time, representing a memory term of the model. Under suitable Lipschitz continuity assumptions on the reaction term and on the nonlocal diffusion coefficient, we prove the existence of a unique solution using a fixed point approach based on Banach's contraction principle. Also, under weaker conditions, we prove the existence of solutions using Darbo's fixed point theorem and compactness arguments. Our analysis relies on specific function spaces, the method of generalized Fourier series, and properties of the solution operator associated with the parabolic differential operator. We provide concrete examples of nonlocal diffusion coefficients, highlighting the applicability of the results. (Joint work with David Brumar)

☜ back to Program

Localization of critical points in conical sets via the method of Nehari manifolds

Andrei Stan

Babeș-Bolyai University and Tiberiu Popoviciu Institute of Romanian Academy, Cluj-Napoca, Romania
andrei.stan@ubbcluj.ro

Using the Nehari manifold method, we establish sufficient conditions such that a smooth functional attains a ground state critical point within an annular domain of a closed cone. This is achieved by constructing a minimization sequence over the Nehari manifold via Ekeland's variational principle and showing that, if the sequence converges, its limit is a critical point of the functional. The localization we obtained immediately allows for multiplicity when applied to disjoint conical sets. To illustrate our results, we consider a two-point boundary value problem, where we obtain a ground state weak solution within a conical shell of a closed cone which is defined in terms of a Harnack inequality with respect to the energy norm. Furthermore, the conditions imposed on the diffusion term naturally extend those from classical examples from literature obtained with the method of Nehari manifold on the entire domain.

back to Program

Considerations on Keller–Segel models with positive total flux

Silvia Frassu [1], Yuya Tanaka [2], and Giuseppe Viglialoro [1]

[1] Dipartimento di Matematica e Informatica, Università degli Studi di Cagliari, Cagliari, Italy
silvia.frassu@unica.it, giuseppe.viglialoro@unica.it
[2] Department of Mathematical Sciences, Kwansei Gakuin University, Nishinomiya, Japan
yuya.tns.6308@gmail.com

Since the advent of the seminal Keller–Segel models describing chemotaxis phenomena involving some cell and chemical distributions, the results obtained for related variants are innumerable. Nevertheless, the common denominator of such studies focuses on the assumption that the equation for the cells obeys a zero-flux boundary condition (impenetrable domains). The aim of this talk is to discuss preliminary results and share considerations on chemotaxis models where the total flux has a positive sign (penetrable domains). This is a joint project with Silvia Frassu and Yuya Tanaka.

References

[1] Frassu, S., Tanaka, Y., Viglialoro, G.; *On a class of Keller–Segel models with positive total flux: analyses and open problems*, work in progress

☜ back to Program

Non-negative solutions of a Dirichlet problem with a non-Lipschitzian term

Fabio Zanolin

Dipartimento di Scienze Matematiche, Informatiche e Fisiche,
Università degli Studi di Udine, Via delle Scienze 2016, 33100 Udine, Italy
fabio.zanolin@uniud.it

We study the existence of solutions, (λ, u), of the problem

$$\begin{cases} -\Delta u = \lambda u - a(x)|u|^{p-1}u & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega, \end{cases}$$

for $0 < p < 1$. Let σ_1 be the smallest eigenvalue of $-\Delta$ in Ω under Dirichlet boundary conditions on $\partial\Omega$. It is proved that there is a component of solutions bifurcating from (σ_1, ∞), unbounded outside of a neighborhood of (σ_1, ∞), and having $u \gneq 0$. This non-negativity for u cannot be improved as is shown via a detailed analysis of the simplest autonomous one-dimensional version of the problem: its set of non-negative solutions possesses a countable set of components, each of them consisting of positive solutions with a fixed (arbitrary) number of bumps. The structure of these components is fully described. These resuls are based on a joint work with J. López-Gómez and P.H. Rabinowitz [1]

References

[1] López-Gómez, J. & Rabinowitz, P.H. & Zanolin, F.; *Non-negative solutions of a sublinear elliptic problem*, J. Fixed Point Theory Appl., vol. 26 number 3 (2024), 32 pages.

☜ back to Program